ECE 158B, Spring 2000, Homework #3

Due the last day of class, or at the very latest, by the beginning of the review session at 10am on Saturday June 8, 2000 (place to be announced).

1. Suppose we are given a fixed rate server with transmission capacity $C = 1$. The arrival rate function is $r_{in}(t)$. Define the unit pulse $p(t)$ as

$$p(t) = \begin{cases} 1 & \text{if } 0 \le t < 1 \\ 0 & \text{otherwise} \end{cases}$$

   Plot the departure process $R_{out}(t)$, the backlog $B(t)$, and the virtual delay $D(t)$ versus t for the various cases below:

   (a) $r_{in}(t) = 3p(t)$

   (b) $r_{in}(t) = 3p(t) + 2p(t-1) + 2p(t-3)$

   (c) $r_{in}(t) = 3p(t) + 2p(t-1) + 2p(t-6)$

2. Suppose we are given a rate function r, such that

$$r(t) = \begin{cases} 0 & \text{, if } t < 0 \\ 1 & \text{, if } 0 \le t < 5 \\ 0 & \text{, if } 5 \le t < 7 \\ 1 & \text{, if } 7 \le t < 15 \\ 0 & \text{, if } 15 \le t \end{cases}$$

   Find the smallest process E such that a corresponding process R has envelope E. In other words, find the empirical envelope of the data stream.

3. A traffic stream arrives to a network element that delivers minimum service curve $S(x) = \rho(x-T)^+$, where $x^+ = \max\{x, 0\}$. The arrival stream has envelope E_{in}, where $E_{in}(x) = M + \min\{Cx, \sigma + \rho x\}$ for $x \ge 0$ and $E_{in}(x) = 0$ for $x < 0$. Assume that $0 < \rho \le C$, $M > 0$, and $\sigma \ge 0$.

   (a) Find an upper bound on the backlog $B(t)$ in terms of σ, ρ, C, M, and T.

   (b) Find an upper bound on the virtual delay $D(t)$ in terms of σ, ρ, C, M, and T.

(c) Let R_1 be the departure process from the network element. Find E_{out} such that R_1 conforms to E_{out}.

(d) Suppose the traffic departing the network element enters a second network element that delivers the same minimum service curve S. Find an upper bound to the virtual delay $D_2(t)$ suffered at the second network element, an upper bound to the backlog $B_2(t)$ of the second network element, and an upper bound to the total virtual end-to-end delay $D_{total}(t)$.

4. Let $S_1(x) = 6(x-1)^+$ and $S_2(x) = (\min\{10(x-2), 5+5(x-2)\})^+$, where we use the notation $x^+ = \max\{x, 0\}$. Find and plot $S_1 * S_2(x)$ versus x.