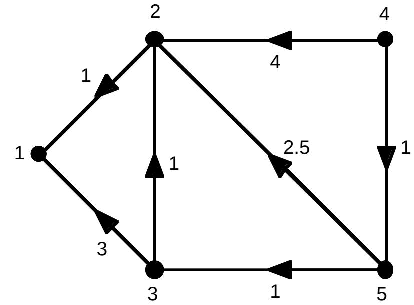
ECE 158B Final, Spring 1997

1. (20 pts) For the weighted directed graph shown, use Dijkstra's algorithm to find the *lengths* of the shortest paths from each node $n = 2, 3, 4, 5$ to the destination node 1. In addition, darken the appropriate edges which lie on a shortest path to the destination node 1 directly on the graph below.



2. (20 pts) Consider Bellman's iteration corresponding to the graph of problem 1. Specifically, given a five dimensional vector $\vec{x} = (x_1, x_2, x_3, x_4, x_5)$, one iteration yields the five dimensional vector $\vec{y} = \vec{f}(\vec{x}) = (y_1, y_2, y_3, y_4, y_5)$, which is used as the input vector for the next iteration, where $y_1 = 0$, $y_2 = 1 + x_1$, $y_3 = \min\{3 + x_1, 1 + x_2\}$, $y_4 = \min\{4 + x_2, 1 + x_5\}$, and $y_5 = \min\{2.5 + x_2, 1 + x_3\}$. Given an initial condition $\vec{x}_0$ for the first iteration, where it is known that $(-2, -2, -2, -2, -2) \leq \vec{x}_0 \leq (\infty, \infty, \infty, \infty, \infty)$, find an upper bound to the number of iterations required until Bellman's iterations converge to the shortest path length vector. Explain clearly why your upper bound is valid as long as the initial condition satisfies the constraint above.

3. (20 pts) Consider a Selective Repeat Protocol with a window size of 8. Recall that in the Selective Repeat Protocol with a window size of 8, the packets are labeled with sequence numbers from 0 to 15 inclusive. Assume that packets and acknowledgments with no errors arrive in the order they are transmitted. Suppose at some point in time, the sender consecutively sends 8 packets with SN values 10,11,12,13,14,15,0, and 1. The next packet received (without errors) by the sender after this has an RN value of x. Find all possible values of x.

4. (20 pts) Suppose a traffic stream with arrival process R_{in} enters a network element which guarantees it a service curve of S, where $S(x) = \eta(x - T)^+$ for all x, and $T \geq 0$. Further suppose that the arrival process R_{in} conforms to b_{in} where

$$b_{in}(x) = \begin{cases} 0 & \text{, if } x \leq 0 \\ \sigma + Kx^{\alpha} + \rho x & \text{, if } x > 0 \text{ ,} \end{cases}$$

where $\sigma > 0$, $K > 0$, $0.5 < \alpha < 1$, and $0 < \rho < \eta$. Find an upper bound on the virtual delay $D(t)$ of data through the network element.

5. (20 pts) An arrival process to a b_{reg} regulator has rate function $r_{in}(t)$, where

$$b_{reg}(x) = \begin{cases} 10 + 4x & \text{, if } x > 0 \\ 0 & \text{, if } x \leq 0 \text{ ,} \end{cases}$$

and

$$r_{in}(t) = \begin{cases} 0 & \text{, if } t < 0 \\ 5 & \text{, if } 0 \leq t < 12 \\ 0 & \text{, if } 12 \leq t < 14 \\ 5 & \text{, if } 14 \leq t < 26 \\ 0 & \text{, if } 26 \leq t \end{cases}$$

Find and plot the corresponding departure process of the regulator, $R_{out}(t)$, for all t. Label your plot carefully.