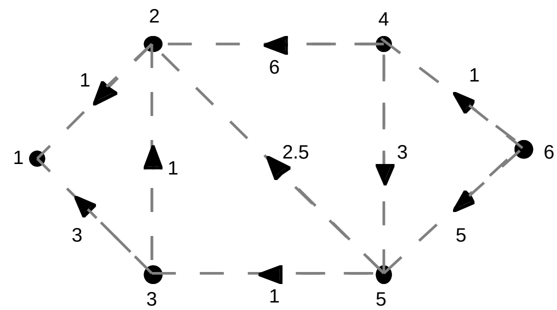
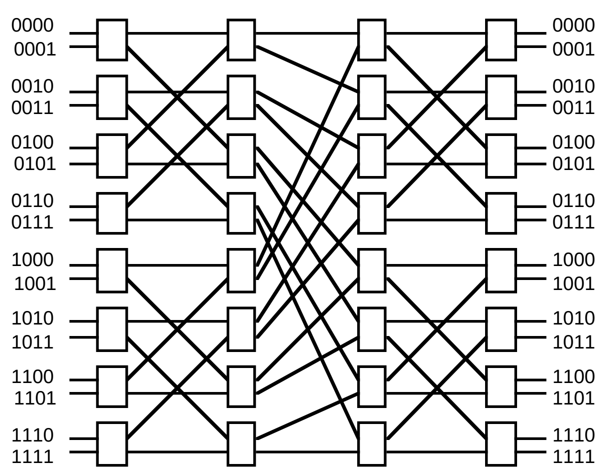
ECE 158B Final, Spring 1998

1. For the weighted directed graph below, use Bellman's iterations to find the lengths of shortest paths each node to node 1, starting from the initial condition $\vec{D}_0 = (D_1^0, D_2^0, \cdots, D_7^0) = (0,0,0,0,0,0,0)$. Document your work clearly. Indicate a shortest path tree directed towards node 1, by darkening the edges if they are in the shortest path tree.



2. A four stage 16 input, 16 output multistage interconnection network is illustrated below. The inputs on the left and the outputs on the right are indexed by binary strings, sequentially from 0 to 15 from top to bottom, as shown. Between the first and second stage, a "banyan" type interconnection pattern is used, which corresponds to "swapping" the least significant bit and the third least significant bit. This same interconnection pattern is used between the third and fourth stage. Between the second and third stage, a shuffle interconnection pattern is used. By considering these interconnection patterns, derive the routing algorithm for this network. That is, given an input $\mathbf{s} = (s_3, s_2, s_1, s_0)$ and an output $\mathbf{d} = (d_3, d_2, d_1, d_0)$, specify how to reach the output $\mathbf{d}$ from the input $\mathbf{s}$. Explain clearly the theory behind your routing algorithm.



3. Suppose R_{in} and R_{out} are the arrival and departure processes to a network element. It is known that $R_{in}(t) = u(t-1) + 2u(t-3)$ and $R_{out}(t) = R_{in}(t-1)$ for all t, where

$$u(x) = \begin{cases} 1 & \text{, if } x \geq 0 \\ 0 & \text{, if } x < 0 \text{ .} \end{cases}$$

Find the backlog $B(t)$ and virtual delay $D(t)$, for all t.

4. Define the process E according to $E(x) = \min\{Cx, \sigma + \rho x\}$ for $x \geq 0$ and $E(x) = 0$ for $x < 0$, where $\sigma > 0$ and $0 < \rho < C$. A traffic stream arrives to a network element that

delivers minimum service curve $S(x) = E(x - T)$, where $T > 0$. This arrival stream has envelope E.

(a) Find an upper bound on the virtual delay $D(t)$ in terms of the given parameters.

(b) Let R_1 be the departure process from the network element. Find an envelope E_1 for R_1 in terms of the given parameters.

(c) Suppose the traffic departing the network element enters a second network element that delivers the same minimum service curve S. In terms of the given parameters, find an upper bound to the virtual delay $D_2(t)$ suffered at the second network element, and an upper bound to the total virtual end-to-end delay $D_{total}(t)$.

5. A traffic stream described by the arrival process R_{in} is incident to a fixed rate server with transmission capacity μ bits/sec. The process R_{in} is known to have envelope E_{in}, where

$$E_{in}(x) = \begin{cases} \lambda x + \alpha x^{\beta} & \text{, if } x > 0 \\ 0 & \text{, if } x \leq 0 \text{ ,} \end{cases}$$

where λ,α, and β are constants such that $0 < \lambda < \mu$, $\alpha > 0$, and $1/2 < \beta < 1$.

(a) In terms of the constants μ, λ,α, and β, find an upper bound to the maximum backlog in the fixed rate server.

(b) Suppose the departure process of the fixed rate server is denoted by R_{out}. In terms of the constants μ, λ,α, and β, find an envelope E_{out} for the departure process R_{out}.