ECE 158B Homework 2, Spring 1999

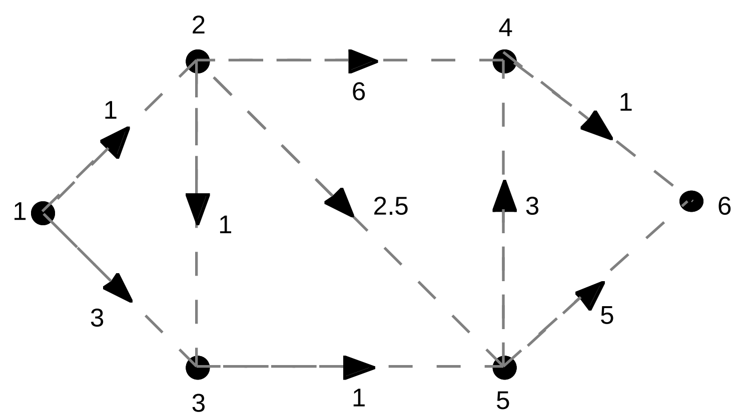
1. A form of the centralized Bellman-Ford routing algorithm is used on a weighted directed graph. There are seven nodes on the graph, labeled 1 through 7, with node 1 being the destination node. The graph may contain cycles of negative or zero weight, and is otherwise unknown. The algorithm starts from some arbitrary intial condition $\vec{D}_0 = (D_1^0, D_2^0, \cdots, D_7^0)$ which has non-negative components (not necessarily the same as in the Bellman-Ford algorithm). On the $k^{th}$ iteration, the value of $\vec{D}_{k+1}$ is obtained from $\vec{D}_k$ by the equation $\vec{D}_{k+1} = \vec{f}(\vec{D}_k)$, where $\vec{f}(\vec{D}) = \vec{D}$ is Bellman's equation, as discussed in class. On the fourth and fifth iterations, the iterates obtained are as follows:

$$\vec{D}_4 = (0, 4, 2, 9, 3, 8, 7)$$
$$\vec{D}_5 = (0, 5, 2, 9, 5, 8, 9)$$

   For each of the cases listed below, state whether or not it is possible to obtain the corresponding value of $\vec{D}_6$ at the sixth iteration, and indicate your reasoning clearly.

   (a) $\vec{D}_6 = (0, 4, 2, 9, 3, 8, 7)$
   (b) $\vec{D}_6 = (0, 5, 2, 9, 5, 8, 9)$
   (c) $\vec{D}_6 = (0, 7, 3, 9, 6, 8, 9)$

2. For the weighted directed graph below, use Dijkstra's algorithm to find the lengths of shortest paths from node 1 to each other node. Document your work clearly. Indicate a shortest path tree emanating from node 1, by darkening the edges if they are in the shortest path tree.




3. Repeat problem 2, but replace each arc in the graph with another arc with the same weight in the opposite direction. Use the centralized Bellman-Ford algorithm to find the lengths of shortest paths to node 1 from each other node. Document your work clearly. Indicate a shortest path tree directed twoards node 1, by darkening the edges if they are in the shortest path tree.

4. Consider the five node graph illustrated below. Notice there is a cycle with zero length in the graph. For this graph, find two distinct solutions to Bellman's equation. In other words, find two *different* 5 dimensional vectors $\vec{x}_1$ and $\vec{x}_2$ such that $\vec{f}(\vec{x}_1) = \vec{x}_1$ and $\vec{f}(\vec{x}_2) = \vec{x}_2$.